

General Form

$$\underline{Ax^2} + Bxy + \underline{Cy^2} + Dx + Ey + F = 0$$

If A and C are nonzero, have the same sign, and are not equal to each other, then the graph is an ellipse.

If A and C are equal and nonzero and have the same sign, the graph is a circle

If A and C are nonzero and have opposite signs, the graph is a hyperbola.

If either A or C is zero, then the graph is a parabola

Identify the graph of each of the following nondegenerate conic sections.

- Ⓐ $4x^2 - 9y^2 + 36x + 36y - 125 = 0$ Hyperbola
- Ⓑ $9y^2 + 16x + 36y - 10 = 0$ Parabola
- Ⓒ $3x^2 + 3y^2 - 2x - 6y - 4 = 0$ Circle
- Ⓓ $-25x^2 - 4y^2 + 100x + 16y + 20 = 0$ Ellipse

For the following exercises, determine which conic section is represented based on the given equation.

6. $9x^2 + 4y^2 + 72x + 36y - 500 = 0$

7. $x^2 - 10x + 4y - 10 = 0$

8. $2x^2 - 2y^2 + 4x - 6y - 2 = 0$

9. $4x^2 - y^2 + 8x - 1 = 0$

10. $4y^2 - 5x + 9y + 1 = 0$

11. $2x^2 + 3y^2 - 8x - 12y + 2 = 0$

12. $4x^2 + 9xy + 4y^2 - 36y - 125 = 0$

13. $3x^2 + 6xy + 3y^2 - 36y - 125 = 0$

14. $-3x^2 + 3\sqrt{3}xy - 4y^2 + 9 = 0$

15. $2x^2 + 4\sqrt{3}xy + 6y^2 - 6x - 3 = 0$

16. $-x^2 + 4\sqrt{2}xy + 2y^2 - 2y + 1 = 0$

17. $8x^2 + 4\sqrt{2}xy + 4y^2 - 10x + 1 = 0$

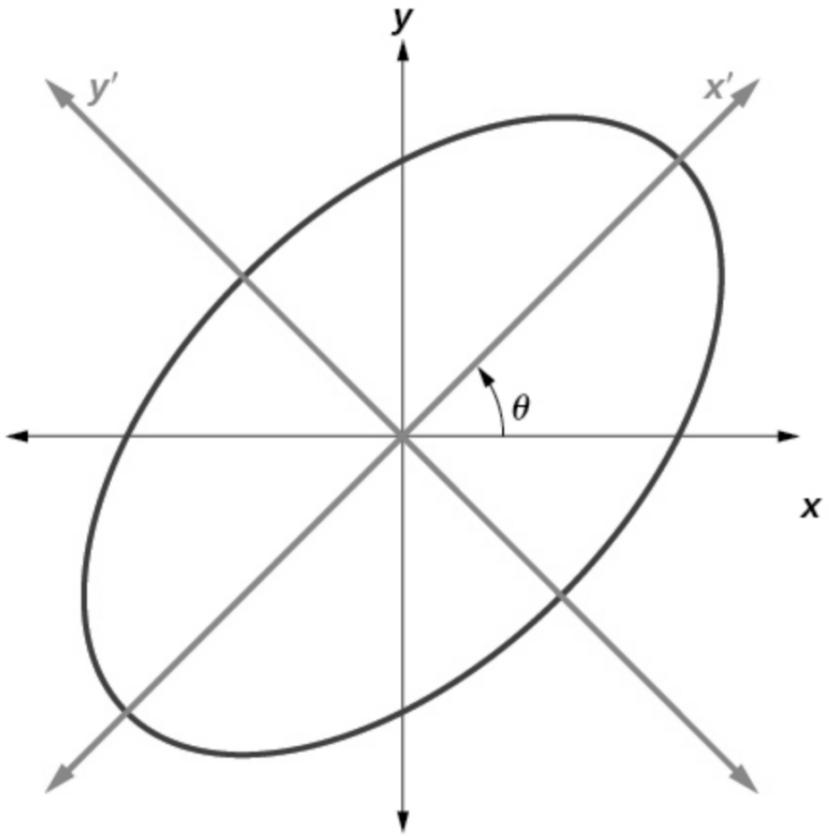


Figure 3 The graph of the rotated ellipse $x^2 + y^2 - xy - 15 = 0$

EQUATIONS OF ROTATION

If a point (x, y) on the Cartesian plane is represented on a new coordinate plane where the axes of rotation are formed by rotating an angle θ from the positive x -axis, then the coordinates of the point with respect to the new axes are (x', y') . We can use the following equations of rotation to define the relationship between (x, y) and (x', y') :

$$\underline{x = x' \cos \theta - y' \sin \theta}$$

and

$$\underline{y = x' \sin \theta + y' \cos \theta}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Finding a New Representation of an Equation after Rotating through a Given Angle

Find a new representation of the equation $2x^2 - xy + 2y^2 - 30 = 0$ after rotating through an angle of $\theta = 45^\circ$.



$$x = x' \cos \theta - y' \sin \theta$$

$$x' \cos 45^\circ - y' \sin 45^\circ$$

$$x' \left(\frac{1}{\sqrt{2}}\right) - y' \left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

$$x = \frac{x' - y'}{\sqrt{2}}$$

$$y = x' \sin \theta + y' \cos \theta$$

$$y = x' \sin 45^\circ + y' \cos 45^\circ$$

$$y = x' \left(\frac{1}{\sqrt{2}}\right) + y' \left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

$$y = \frac{x' + y'}{\sqrt{2}}$$

$$2x^2 - xy - 2y^2 - 30 = 0$$

$$2 \left(\frac{x' - y'}{\sqrt{2}} \right)^2 - \left(\frac{x' - y'}{\sqrt{2}} \right) \left(\frac{x' + y'}{\sqrt{2}} \right) - 2 \left(\frac{x' + y'}{\sqrt{2}} \right)^2 - 30 = 0$$

$$2 \left(\frac{(x' - y')^2}{2} \right) - \frac{(x' - y')(x' + y')}{2} - 2 \left(\frac{(x' + y')^2}{2} \right) - 30 = 0$$

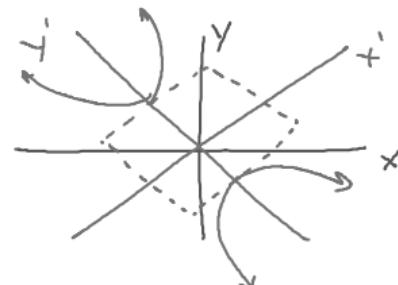
$$(x' - y')^2 - \frac{(x' - y')(x' + y')}{2} - (x' + y')^2 - 30 = 0$$

$$2(x' - y')^2 - (x' - y')(x' + y') - 2(x' + y')^2 - 60 = 0$$

$$2(x'^2 - 2x'y' + y'^2) - (x'^2 - y'^2) - 2(x'^2 + 2x'y' + y'^2) - 60 = 0$$

~~$$2x'^2 - 4x'y' + 2y'^2 - x'^2 + y'^2 - 2x'^2 - 4x'y' - 2y'^2 - 60 = 0$$~~

$$-x'^2 - 8x'y' + y'^2 - 60 = 0$$



Finding a new representation of the equation $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$ after rotating through an angle of 30 degrees.

$$X = x' \cos \theta - y' \sin \theta$$

$$\begin{aligned} X &= x' \cos 30^\circ - y' \sin 30^\circ \\ &= x' \left(\frac{\sqrt{3}}{2}\right) - y' \left(\frac{1}{2}\right) \end{aligned}$$

$$\frac{x'\sqrt{3}}{2} - \frac{y'}{2}$$

$$X = \frac{x'\sqrt{3} - y'}{2}$$

$$7 \left(\frac{x'\sqrt{3} - y'}{2} \right)^2 - 6\sqrt{3} \left(\frac{x'\sqrt{3} - y'}{2} \right) \left(\frac{x' + y'\sqrt{3}}{2} \right) + 13 \left(\frac{x' + y'\sqrt{3}}{2} \right)^2 - 16$$

$$7 \left(\frac{(x'\sqrt{3} - y')^2}{4} \right) - 6\sqrt{3} \left(\frac{(x'\sqrt{3} - y')(x' + y'\sqrt{3})}{4} \right) + 13 \left(\frac{(x' + y'\sqrt{3})^2}{4} \right) - 16$$

$$7((x'\sqrt{3} - y')^2) - 6\sqrt{3}(x'\sqrt{3} - y)(x' + y'\sqrt{3}) + 13(x' + y'\sqrt{3})^2 - 64 = 0$$

$$7(3x'^2 - 2x'y'\sqrt{3} + y'^2) - 6\sqrt{3}(x'^2\sqrt{3} + 2x'y'\sqrt{3}\sqrt{3}) + 13(x'^2 + 2x'y'\sqrt{3} + 3y'^2) - 64$$

$$\cancel{21x'^2} - \cancel{14x'y'\sqrt{3}} + \cancel{7y'^2} = \cancel{18x'^2} - \cancel{12\sqrt{3}x'y'} + \cancel{18y'^2} + \cancel{13x'^2} + \cancel{26x'y'\sqrt{3}} + \cancel{9y'^2} - 64$$

$$16x'^2 + 64y'^2 - 64 = 0$$

$$\frac{x'^2}{4} + \frac{y'^2}{1} = 1$$

$$\frac{25}{39} \frac{39}{64}$$

